

BARI-TH/96-236  
 CERN-TH/96-125  
 hep-ph/9605274  
 May 1996

## Measuring $\alpha_s(Q^2)$ in $\tau$ Decays

Maria Girone

*Dipartimento di Fisica, INFN Sezione di Bari, 70126 Bari, Italy*

and

Matthias Neubert

*Theory Division, CERN, CH-1211 Geneva 23, Switzerland*

### Abstract

The decay rate of the  $\tau$  lepton into hadrons of invariant mass smaller than  $Q \gg \Lambda_{\text{QCD}}$  can be calculated in QCD using the OPE. Using experimental data on the hadronic mass distribution, the running coupling constant  $\alpha_s(Q^2)$  is extracted in the range  $0.85 \text{ GeV} < Q < m_\tau$ , where its value changes by about a factor 2. At  $Q = m_\tau$ , the result is  $\alpha_s(m_\tau^2) = 0.33 \pm 0.03$ , corresponding to  $\alpha_s(m_Z^2) = 0.119 \pm 0.004$ . The running of the coupling constant is in excellent agreement with the QCD prediction based on the three-loop  $\beta$ -function.

*To appear in the Proceedings of  
 Les Rencontres de Physique de la Vallée d'Aoste  
 (La Thuile, Italy, March 1996)*

and

*Second Workshop on Continuous Advances in QCD  
 (Minneapolis, Minnesota, March 1996)*

# 1 Introduction

One of the most accurate methods to determine  $\alpha_s$  in the low-energy region is provided by the measurement of  $R_\tau$ , the  $\tau$  decay rate into hadrons normalized to the leptonic decay rate:

$$R_\tau = \frac{\Gamma(\tau \rightarrow \nu_\tau + \text{hadrons})}{\Gamma(\tau \rightarrow \nu_\tau e \bar{\nu}_e)}. \quad (1)$$

$R_\tau$  can be calculated in QCD using the Operator Product Expansion (OPE) [1, 2]. The result is:

$$\begin{aligned} R_\tau &= N_c \left\{ 1 + \delta_{\text{pert}}[\alpha_s(m_\tau^2)] + \delta_{\text{power}} \right\} \\ &= N_c \left\{ 1 + \frac{\alpha_s(m_\tau^2)}{\pi} + 5.202 \left( \frac{\alpha_s(m_\tau^2)}{\pi} \right)^2 + 26.37 \left( \frac{\alpha_s(m_\tau^2)}{\pi} \right)^3 + \dots \right. \\ &\quad \left. - 8 |V_{us}|^2 \frac{m_s^2}{m_\tau^2} + 32\pi^2 \frac{\langle m\bar{\psi}\psi \rangle}{m_\tau^4} - 2 \frac{\langle O_6 \rangle}{m_\tau^6} + \dots \right\}. \end{aligned} \quad (2)$$

The non-perturbative power corrections in this expression are proportional to the strange-quark mass, the quark condensate, and higher-dimensional condensates. Because all contributions of dimension less than six vanish in the chiral limit, the power corrections are numerically small; using standard values of the QCD parameters, one finds  $\delta_{\text{power}} = -(1.4 \pm 0.5)\%$ . This, together with the fact that the perturbation series is known to third order, make  $R_\tau$  a good observable to measure  $\alpha_s$ .

Experimentally,  $R_\tau$  is obtained from the relation  $R_\tau = 1/B_e - 1.97256$ , where  $B_e$  is the leptonic branching ratio. Direct measurements give  $B_e = (17.80 \pm 0.06)\%$  [3], whereas using the  $\tau$  lifetime,  $\tau_\tau = (291.3 \pm 1.6)$  fs [4], we obtain  $B_e = \tau_\tau/\tau_\mu (m_\tau/m_\mu)^5 = (17.84 \pm 0.10)\%$ . Averaging the two results gives  $R_\tau = 3.642 \pm 0.010$ , and taking into account small electroweak radiative corrections not displayed in (2) we obtain

$$\delta_{\text{pert}}[\alpha_s(m_\tau^2)] = 0.205 \pm 0.003_{\text{exp}} \pm 0.005_{\text{th}}. \quad (3)$$

The dominant theoretical uncertainty in extracting  $\alpha_s$  from this measurement comes from the truncation of perturbation theory [5], which induces an error of order  $\alpha_s^4$ . This uncertainty can be estimated by considering some approximate resummations of the perturbation series (starting at order  $\alpha_s^4$ ) and comparing them to the fixed-order calculation. The resummation procedure of Le Diberder and Pich [6] resums certain “large- $\pi^2$ ” terms to all orders in perturbation theory. Recently, another class of terms, the so-called renormalon chains [7], have been investigated. These are the terms of order  $\beta_0^{n-1}\alpha_s^n$  in the perturbation series for  $\delta_{\text{pert}}$ , where  $\beta_0$  is the first coefficient of the QCD  $\beta$ -function. The resummation

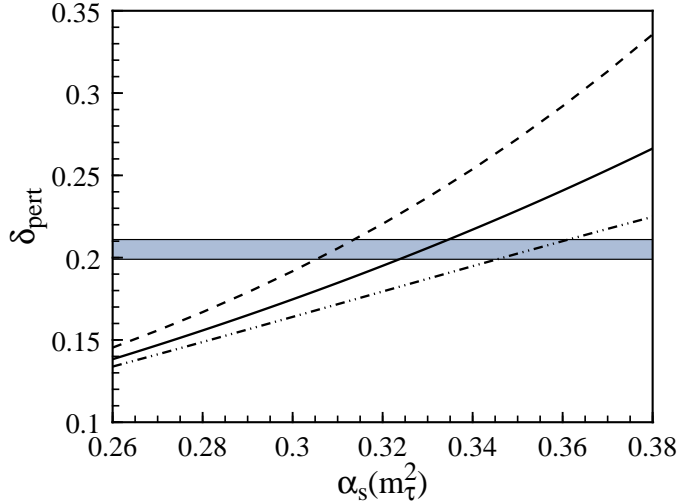


Fig. 1. Different perturbative approximations for the quantity  $\delta_{\text{pert}}$ : exact order- $\alpha_s^3$  result in the  $\overline{\text{MS}}$  scheme (solid line), resummation of renormalon chains (dashed line), resummation of Le Diberder and Pich (dash-dotted line). The experimental result (3) is shown as a band.

of such terms in the case of  $R_\tau$  has been discussed in Refs. [8, 9]. In Fig. 1, we show the corresponding theoretical predictions for  $\delta_{\text{pert}}$  as a function of  $\alpha_s(m_\tau^2)$ . We conclude that  $\delta\alpha_s(m_\tau^2) \simeq \pm 0.03$  is a reasonable estimate of the truncation error. This leads to

$$\alpha_s(m_\tau^2) = 0.33 \pm 0.03, \quad \alpha_s(m_Z^2) = 0.119 \pm 0.004. \quad (4)$$

For the sake of completeness, we have translated our result into a value of  $\alpha_s$  at the mass of the  $Z$  boson.

The analysis just described provides one of the best determination of the QCD coupling constant in the low-energy region. The result (4) is included in Fig. 2, which shows a collection of measurements of  $\alpha_s$  performed at different energy scales [10]. Besides  $\tau$  decays, low-energy ( $Q \sim 1.6\text{--}10$  GeV) measurements come from deep-inelastic scattering and  $\Upsilon$  spectroscopy and decays. At higher energies ( $Q \sim 30\text{--}130$  GeV), the most reliable determinations of  $\alpha_s$  come from measurements of the total cross section, jet rates and event shapes in  $e^+e^-$ ,  $p\bar{p}$  and  $ep$  collisions. Taken all together, these measurements provide clear evidence for the “running” of the effective coupling constant  $\alpha_s(Q^2)$ , which in QCD is predicted to decrease with the momentum transfer. This property of “asymptotic freedom” [11] is one of the key predictions of QCD. Formally, it is expressed by the fact that the  $\beta$ -function is positive, where

$$\frac{d\alpha_s(Q^2)}{d \ln Q^2} = -\alpha_s(Q^2) \beta[\alpha_s(Q^2)],$$

$$\beta(\alpha_s) = \beta_0 \frac{\alpha_s}{4\pi} + \beta_1 \left(\frac{\alpha_s}{4\pi}\right)^2 + \beta_2 \left(\frac{\alpha_s}{4\pi}\right)^3 + \dots, \quad (5)$$

and  $\beta_0 = 9$ ,  $\beta_1 = 64$  and  $\beta_2 = 3863/6$  are the first three expansion coefficients of the  $\beta$ -function, evaluated for  $n_f = 3$  light quark flavours. (The value of  $\beta_2$  is specific to the  $\overline{\text{MS}}$  renormalization scheme.)

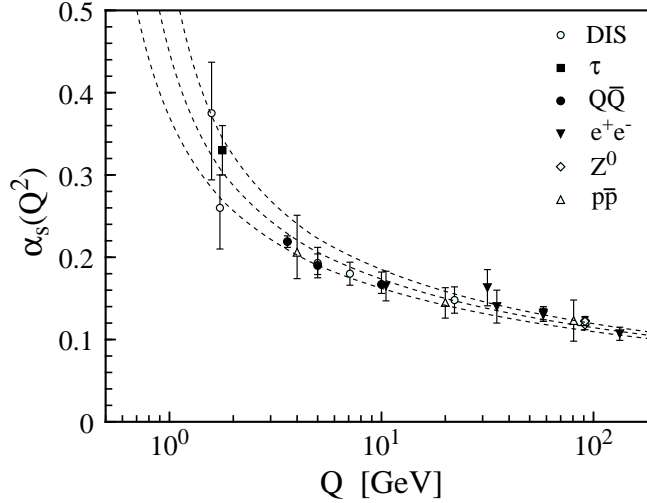


Fig. 2. Compilation of  $\alpha_s$  measurements. The curves correspond to the QCD prediction for the running coupling constant for  $\alpha_s(m_Z^2) = 0.116 \pm 0.005$ .

A test of the running of  $\alpha_s$  by combining measurements performed in many experiments operating at different energy scales has the disadvantage of involving different experimental systematic errors, as well as different levels of sophistication of the theoretical calculations. Therefore, it is an appealing idea to measure the scale dependence of the coupling constant in a single experiment. This can be done in high-energy experiments at  $p\bar{p}$  and  $ep$  colliders, where a large range of  $Q$  values can be probed simultaneously [12]. However, so far the precision obtained in these measurements is rather low. In this talk, we propose a high-precision test of the running of  $\alpha_s$  in the low-energy region ( $0.85 \text{ GeV} < Q < m_\tau$ ), using data obtained in a single experiment [13]. The value of  $\alpha_s$  changes by about a factor 2 in this energy range, which is equivalent to the variation between 5 and 100 GeV.

## 2 Extraction of $\alpha_s(Q^2)$ in $\tau$ decays

We shall consider the  $\tau$  decay rate into hadrons of invariant mass squared smaller than  $s_0$ , normalized to the leptonic decay rate:

$$R_\tau(s_0) = \frac{\Gamma(\tau \rightarrow \nu_\tau + \text{hadrons}; s_{\text{had}} < s_0)}{\Gamma(\tau \rightarrow \nu_\tau e \bar{\nu}_e)} = \int_0^{s_0} ds \frac{dR_\tau(s)}{ds}, \quad (6)$$

where  $dR_\tau/ds$  is the inclusive hadronic spectrum, which has been measured by the CLEO and ALEPH Collaborations [14, 15]. To obtain  $dR_\tau/ds$ , we have multiplied the normalized distributions by  $R_\tau$ . The result is shown in the upper portion of Fig. 3. Not shown in the figure is the contribution from  $\tau \rightarrow h^- \nu_\tau$  with  $h^- = \pi^-$  or  $K^-$ , which has a branching ratio of  $(11.77 \pm 0.14)\%$  [3]. Integrating these spectra over  $s$  and combining the results weighted by their statistical errors, we obtain the distribution  $R_\tau(s_0)$  shown in the lower portion of the figure. Systematic errors have been estimated by taking the difference between the CLEO and ALEPH data, and added in quadrature with the statistical errors. Since the errors are strongly correlated, the result is presented as a band.

Using the analyticity properties of QCD spectral functions, the quantity  $R_\tau(s_0)$  can be represented as a contour integral along a circle of radius  $|s| = s_0$  in the complex  $s$ -plane (for simplicity, we quote the result in the chiral limit):

$$R_\tau(s_0) = \frac{1}{2\pi i} \oint_{|s|=s_0} \frac{ds}{s} w\left(\frac{s_0}{m_\tau^2}, \frac{s}{m_\tau^2}\right) D(s). \quad (7)$$

Here

$$w(x, y) = 2(x - y) - 2(x^3 - y^3) + (x^4 - y^4) \quad (8)$$

is the phase-space function, and  $D(s)$  is a current-current correlation function, which contains all QCD dynamics. The representation (7) shows that  $s_0$  is the only scale at which QCD dynamics is probed; the  $\tau$ -lepton mass appears only in the phase space. Provided that  $s_0 \gg \Lambda_{\text{QCD}}^2$ , the correlation function  $D(s)$  is needed at large momentum transfer only, and the OPE can be employed to calculate  $R_\tau(s_0)$  as a function of  $\alpha_s(s_0)$  and  $x_0 = s_0/m_\tau^2$ :

$$R_\tau(s_0) = N_c \left\{ r_{\text{pert}}[\alpha_s(s_0), x_0] + r_{\text{power}}(x_0) \right\}. \quad (9)$$

The perturbative contribution is given by ( $a_0 \equiv \alpha_s(s_0)/\pi$ ):

$$\begin{aligned} r_{\text{pert}}[\alpha_s(s_0), x_0] = & (2x_0 - 2x_0^3 + x_0^4) \left[ 1 + a_0 + 1.640a_0^2 - 10.28a_0^3 + (K_4 - 156.0)a_0^4 \right] \\ & + (2x_0 - \frac{2}{3}x_0^3 + \frac{1}{4}x_0^4) (2.25a_0^2 + 11.38a_0^3 - 46.24a_0^4) \\ & + (2x_0 - \frac{2}{9}x_0^3 + \frac{1}{16}x_0^4) (10.125a_0^3 + 94.81a_0^4) \\ & + (2x_0 - \frac{2}{27}x_0^3 + \frac{1}{64}x_0^4) 68.34a_0^4 + O(a_0^5), \end{aligned} \quad (10)$$

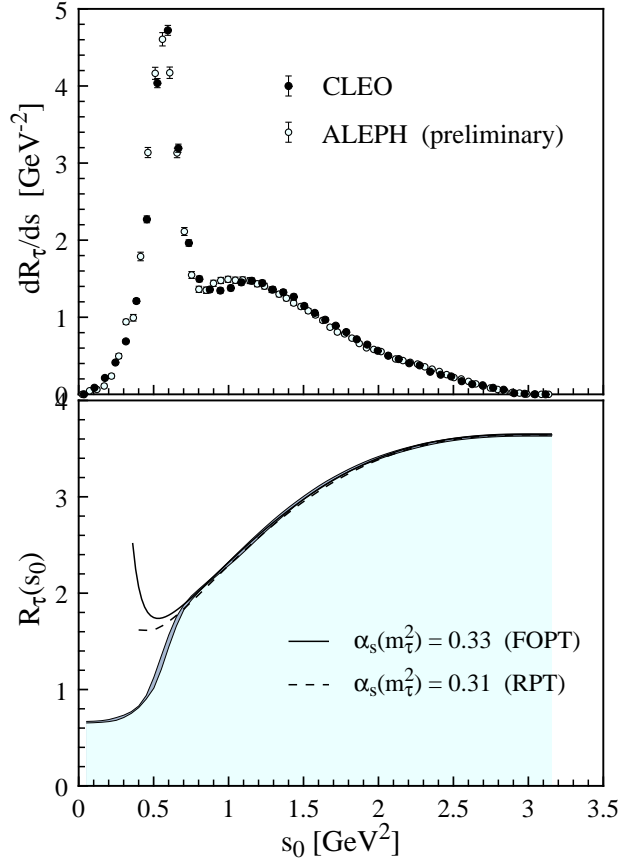


Fig. 3. Upper plot: The hadronic mass distribution  $dR_\tau/ds$  in  $\tau$  decays. Shown are the statistical errors after correcting for detector effects. CLEO data from electron-tagged and muon-tagged events have been combined. The ALEPH data are preliminary. Lower plot: The integrated spectrum  $R_\tau(s_0)$ . The experimental result, including statistical and systematic errors, is presented as a band. The curves will be explained in Sect. 3.

where  $K_4$  is the five-loop coefficient in the Adler function, which is currently not known exactly. In our analysis, we use the estimate  $K_4 \simeq 27.5$  [16] obtained using the methods of Ref. [17]. As in the case of  $R_\tau$ , the truncation of the perturbation series will turn out to be the main theoretical uncertainty in our analysis. We estimate the importance of the unknown higher-order contributions (of order  $a_0^5$  and higher) by resumming the renormalon-chain contributions to all orders in perturbation theory, using the results of Ref. [9]. We shall compare fixed-order perturbation theory with this resummation and take the difference as an estimate of the perturbative uncertainty. This estimate of the truncation error is more conservative than that obtained by dropping the last term in the series in (10).

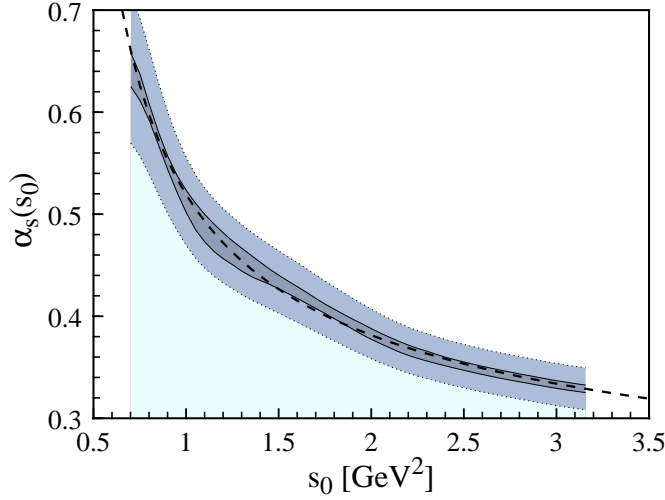


Fig. 4. Values of  $\alpha_s(s_0)$  extracted from the data on  $R_\tau(s_0)$ . The dark band represents the experimental errors, the light one the sum of the experimental and theoretical errors. The errors are strongly correlated. The dashed line shows the three-loop QCD prediction for the running coupling constant.

The power corrections in (9) are given by

$$r_{\text{power}}(x_0) = -6 |V_{us}|^2 (1 + x_0 - x_0^2 + \frac{1}{3}x_0^3) \frac{m_s^2}{m_\tau^2} + 32\pi^2 \frac{\langle m\bar{\psi}\psi \rangle}{m_\tau^4} - 2 \frac{\langle O_6 \rangle}{m_\tau^6} + \dots \quad (11)$$

Note that, as a simple consequence of the representation (7), no inverse powers of  $s_0$  appear in this expression [13]. This is true as long as the coefficients of the power corrections to the correlation function  $D(s)$  do not contain logarithms of  $s$ . As a result, the OPE converges well down to low scales  $s_0$ . For instance, we find  $r_{\text{power}} = -(1.4 \pm 0.5)\%$  at  $s_0 = m_\tau^2$ , and  $r_{\text{power}} = -(1.5 \pm 0.5)\%$  at  $s_0 = 1 \text{ GeV}^2$ . The break-down of the OPE (see Sect. 3 below) will thus not be driven by a blow-up of the series of power corrections. Another important feature of (11) is that the terms involving the vacuum condensates are independent of  $s_0$ . Hence, the uncertainties in the values of the condensates do not affect the  $s_0$  dependence of  $R_\tau(s_0)$ , which will be used to study the running of  $\alpha_s(s_0)$ .

From the measurement of the quantity  $R_\tau(s_0)$  shown in Fig. 3, we extract  $\alpha_s(s_0)$  as a function of  $s_0$  by fitting to the data the theoretical prediction obtained using fixed-order perturbation theory. The result, including experimental errors only, is represented by the dark band in Fig. 4. Theoretical uncertainties arise from the truncation of the perturbation series and from the uncertainty in the values of the nonperturbative parameters. They affect the overall scale of the  $\alpha_s$  values (by about 8–10%), but have very little effect on the evolution of the coupling constant. The sum of the experimental and theoretical errors is represented by the light band. The dashed curve shows the QCD predictions for

$\alpha_s(s_0)$  obtained at three-loop order, normalized to the central value of the data at  $s_0 = m_\tau^2$ . The observed scale dependence of the running coupling constant is in excellent agreement with the QCD prediction.

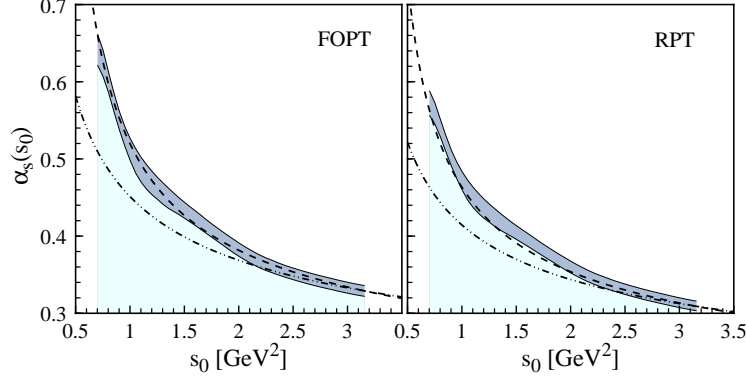


Fig. 5. Values of  $\alpha_s(s_0)$  extracted from the data on  $R_\tau(s_0)$  using fixed-order (FOPT) and resummed perturbative theory (RPT). The dashed lines show the QCD prediction obtained using the three-loop  $\beta$ -function. The dash-dotted lines refer to the one-loop  $\beta$ -function.

As mentioned above, the main theoretical uncertainty comes from the truncation of the perturbation series. This is illustrated in Fig. 5, where the evolution of  $\alpha_s(s_0)$  as a function of  $s_0$  is shown separately for fixed-order and resummed perturbation theory. The curves show the QCD predictions for the running coupling constant obtained at one- and three-loop order, normalized to the data at  $s_0 = m_\tau^2$ . It is seen that higher-order corrections effectively renormalize the overall scale of the  $\alpha_s$  values (i.e. the  $\Lambda_{\text{QCD}}$  parameter). For instance, the value of  $\alpha_s$  at  $s_0 = m_\tau^2$  changes from 0.33 (fixed-order) to 0.31 (resummed). The difference between the two results for  $\alpha_s(s_0)$  has been used to estimate the truncation error.

Fig. 6 shows our result combined with the other measurements of  $\alpha_s$  collected in Fig. 2. We have replaced the data point at  $Q = m_\tau$  by the band shown in Fig. 4, which extends to much lower values of  $Q$ . This figure demonstrates nicely the main features of our approach: it extends the range of  $\alpha_s$  values accessible to experiments, thus allowing a measurement of the strong coupling constant at scales lower than the lowest ones attainable before. Moreover, it provides a test of the QCD evolution of  $\alpha_s$  with higher precision than all other single measurements of the running to date.



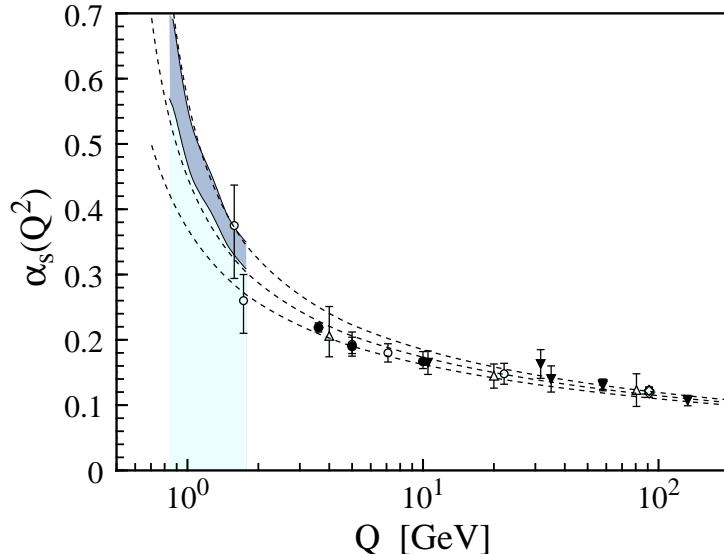


Fig. 6. Compilation of  $\alpha_s$  measurements including our result obtained from the analysis of hadronic  $\tau$  decays. The curves correspond to the QCD prediction for the running coupling constant for  $\alpha_s(m_Z^2) = 0.121, 0.116, 0.111$  (top to bottom).

### 3 Break-down of the OPE and quark–hadron duality

An important question which we have to address is to determine the lowest value of  $s_0$  for which our analysis can be trusted. In other words, at which point do we expect the OPE to break down? To answer this question, it is important to realize that we are applying the OPE in the physical region (i.e. the region of time-like momenta), where QCD cannot be used to calculate correlation functions such as  $D(s)$ . The reason why we trust the calculation of  $R_\tau(s_0)$  is that to perform the contour integral in (7) requires knowledge of the correlation function for large (complex) momenta only. Moreover, the integrand vanishes for  $s = s_0$ , where the contour touches the branch cut of  $D(s)$ ; hence, the main contributions come from regions far away from the singularities, where the OPE can be applied. Another way to say this is that in the calculation of  $R_\tau(s_0)$  we assume quark–hadron duality, which is the hypothesis that QCD can be employed to calculate physical decay rates if they are “smeared” over a sufficiently wide energy interval [18]. In the present case, this smearing is provided by the integration over the range  $0 < s < s_0$  in (6). The question of how accurate the duality assumption is and for what values of  $s_0$  it applies is, however, a phenomenological one. Despite of some interesting new ideas [19], it cannot be answered yet from theoretical grounds.

To test the assumption of duality, we compare the data for the quantity

$R_\tau(s_0)$  with the theoretical predictions obtained from the OPE, using both fixed-order and resummed perturbation theory. The results are shown by the two curves in the lower portion of Fig. 3. In obtaining these curves, we have adjusted the value of  $\alpha_s(m_\tau^2)$  so as to fit the data at  $s_0 = m_\tau^2$ . The value of  $\alpha_s(s_0)$  is then obtained from the solution of the renormalization-group equation (5). Theoretical uncertainties have little influence on the  $s_0$  dependence of  $R_\tau(s_0)$ . For the perturbative part of the calculation, this is apparent from the good agreement of the two theoretical curves in Fig. 3, which refer to values of  $\alpha_s(m_\tau^2)$  that differ by 9%. Hence, the  $s_0$  dependence of  $R_\tau(s_0)$  is predicted essentially without any free parameters, and the comparison of the data with the theoretical predictions provides a direct test of quark–hadron duality.

We find excellent agreement over the range  $0.7 \text{ GeV}^2 < s_0 < m_\tau^2$ , indicating that in  $\tau$  decays duality holds as soon as the integral over the hadronic mass distribution includes the  $\rho$  resonance peak. This justifies *a posteriori* our choice of the energy interval in the previous section. It is remarkable that, once  $s_0$  exceeds the value of  $0.7 \text{ GeV}^2$ , the onset of duality happens almost instantaneously. Since the  $\rho$  meson is such a prominent resonance, this is the best possible scenario that could be expected. The small oscillation of the experimental band around the theoretical curve, which could be due to some deviations from duality in the region of the  $a_1$  resonance, are not significant given the precision of the data. Even if such oscillations will be confirmed in further analyses based on more precise data, they will clearly not put a severe limitation on the applicability of our method.

## 4 Measurement of the $\beta$ -function

To quantify the agreement between the data and the QCD prediction for the running coupling constant exhibited in Figs. 4 and 5, we extract from the data the  $\beta$ -function defined in (5) and compare the result to the prediction of QCD perturbation theory. Introducing the variable  $x = \alpha_s(s_0)/4\pi$ , we have

$$-\frac{4\pi}{\alpha_s^2(s_0)} \frac{d\alpha_s(s_0)}{d \ln s_0} = \frac{\beta(x)}{x} = \beta_0 + \beta_1 x + \beta_2 x^2 + \dots \quad (12)$$

We approximate the derivative  $d\alpha_s/d \ln s_0$  by a ratio of differences,  $\Delta\alpha_s/\Delta \ln s_0$ , for a selected set of  $s_0$  values chosen such that the differences  $\Delta\alpha_s$  are large enough to be significant given the errors in the measurement. For  $\alpha_s(s_0)$  in (12) we take the central value of each interval. We use the following  $s_0$  values: 0.75, 0.95, 1.35, 2.06, and  $3.16 \text{ GeV}^2$ , corresponding to four intervals of increasing width  $\Delta \ln s_0$ , but constant  $\Delta\alpha_s \simeq 0.075$ . The results are shown in Fig. 7. The circles are obtained using fixed-order perturbation theory, while the squares refer to resummed perturbation theory. As expected, the two methods give very similar results for the running of the coupling constant. The estimate of the

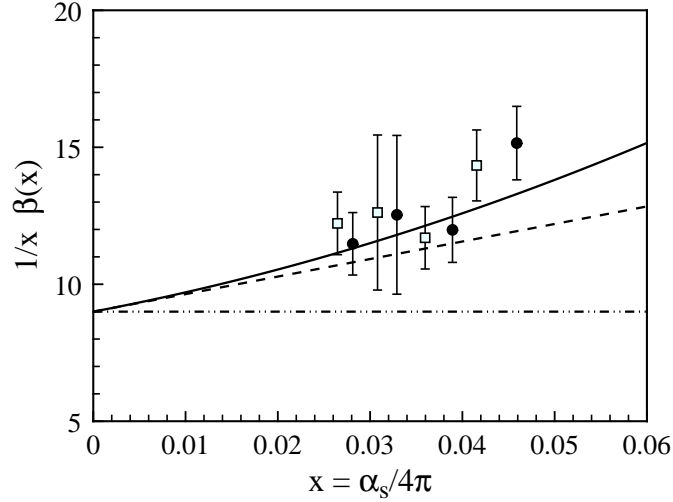


Fig. 7. Experimental determination of the  $\beta$ -function. The circles are obtained using fixed-order perturbation theory, the squares refer to resummed perturbation theory. The curves show the QCD  $\beta$ -function at one-loop (dash-dotted), two-loop (dashed) and three-loop (solid) order.

errors includes the theoretical uncertainties, the error due to the choice of finite intervals in  $\alpha_s$ , and the experimental errors, which in this case are the dominant ones. The curves in Fig. 7 show the QCD  $\beta$ -function at one-, two- and three-loop order in perturbation theory. The data provide clear evidence for the running of the coupling constant. Moreover, they prefer a running that is stronger than predicted at one-loop order. Indeed, between the three curves, the best description of the data is provided by the three-loop prediction. Performing a fit with the three-loop  $\beta$ -function, where  $\beta_0 = 9$  and  $\beta_1 = 64$  are kept fixed but the three-loop coefficient  $\beta_2$  is treated as a parameter, we find  $\beta_2^{\text{exp}}/\beta_2^{\text{th}} = 1.6 \pm 0.7$  using fixed-order perturbation theory, and  $1.8 \pm 0.8$  using resummed perturbation theory.

We believe that such an experimental determination of the  $\beta$ -function beyond the leading order can at present be done only in  $\tau$  decays. (A high-precision measurement of  $R_{e^+e^-}(s)$  in the region below the charmonium resonances would provide an alternative place for such a study.) At higher energies, the value of  $\alpha_s$  is too small to distinguish between the three curves in Fig. 7; measurements in the region  $Q \sim 100$  GeV, for instance, correspond to values  $x \sim 0.01$ .

## 5 Conclusions

We have presented a method to measure the running coupling constant  $\alpha_s(Q^2)$  in the low-energy region  $0.85 \text{ GeV} < Q < m_\tau$ , using  $\tau$ -decay data obtained in a

single experiment. At  $Q = m_\tau$ , we obtain  $\alpha_s(m_\tau^2) = 0.33 \pm 0.03$ , corresponding to the rather precise value  $\alpha_s(m_Z^2) = 0.119 \pm 0.004$ . Our method provides a test of the scale dependence of the coupling constant in a region where this effect is most pronounced. The theoretical analysis is based on the OPE and the assumption of quark–hadron duality. We have tested this assumption and find that it holds provided the  $\tau$  decay rate is integrated over an energy interval large enough to include the  $\rho$  resonance peak. Our analysis provides a test of QCD at scales comparable with the lowest ones attainable before ( $Q \simeq 1.6$  GeV in deep-inelastic scattering), and with higher precision than all other single measurements of the running to date. We have extracted for the first time the  $\beta$ -function from data and find that it is in good agreement with the three-loop prediction of QCD.

## References

- [1] M.A. Shifman, A.I. Vainshtein and V.I. Zakharov, Nucl. Phys. B **147**, 385 and 448 (1979).
- [2] E. Braaten, S. Narison and A. Pich, Nucl. Phys. B **373**, 581 (1992).
- [3] G. Rahal-Callot, to appear in: Proc. Int. Europhysics Conf. on High Energy Physics, Brussels, Belgium, September 1995.
- [4] S.R. Wasserbaech, to appear in: Proc. Workshop on the Tau–Charm Factory, Argonne, Illinois, June 1995.
- [5] G. Altarelli, P. Nason and G. Ridolfi, Z. Phys. C **68**, 257 (1995).
- [6] F. Le Diberder and A. Pich, Phys. Lett. B **286**, 147 (1992); **289**, 165 (1992).
- [7] G. 't Hooft, in: Proc. 15th Int. School of Subnuclear Physics, Erice, Sicily, 1977, ed. A. Zichichi (Plenum Press, New York, 1979), p. 943; B. Lautrup, Phys. Lett. B **69**, 109 (1977); G. Parisi, Phys. Lett. B **76**, 65 (1978); Nucl. Phys. B **150**, 163 (1979); F. David, Nucl. Phys. B **234**, 237 (1984); **263**, 637 (1986); A.H. Mueller, Nucl. Phys. B **250**, 327 (1985); V.I. Zakharov, Nucl. Phys. B **385**, 452 (1992); M. Beneke and V.I. Zakharov, Phys. Rev. Lett. **69**, 2472 (1992); D. Broadhurst, Z. Phys. C **58**, 339 (1993).
- [8] P. Ball, M. Beneke and V.M. Braun, Nucl. Phys. B **452**, 563 (1995); C.N. Lovett-Turner and C.J. Maxwell, Nucl. Phys. B **452**, 188 (1995).
- [9] M. Neubert, Nucl. Phys. B **463**, 511 (1996).
- [10] B.R. Webber, in: Proc. 27th Int. Conf. on High Energy Physics, Glasgow, Scotland, July 1994, eds. P.J. Bussey and I.G. Knowles (IOP Publ., Bristol, 1995), Vol. 1, p. 213; R.K. Ellis and B.R. Webber, private communication.

- [11] D.J. Gross and F. Wilczek, Phys. Rev. Lett. **30**, 1343 (1973); H.D. Politzer, Phys. Rev. Lett. **30**, 1346 (1973).
- [12] M. Derrick et al. (ZEUS Collaboration), Phys. Lett. B **363**, 201 (1995); W.T. Giele, E.W.N. Glover and J. Yu, FERMILAB-Pub-95/127-T (1995) [hep-ph/9506442].
- [13] M. Girone and M. Neubert, Phys. Rev. Lett. **76**, 3061 (1996).
- [14] T. Coan et al. (CLEO Collaboration), Phys. Lett. B **356**, 580 (1995).
- [15] L. DufLOT, in: Proc. 3rd Workshop on Tau Lepton Physics, ed. L. Rolandi, Nucl. Phys. B (Proc. Suppl.) **40**, 37 (1995).
- [16] A.L. Kataev and V.V. Starshenko, Mod. Phys. Lett. A **10**, 235 (1995).
- [17] P.M. Stevenson, Phys. Rev. D **23**, 2916 (1981); G. Grunberg, Phys. Lett. B **221**, 70 (1980); Phys. Rev. D **29**, 2315 (1984).
- [18] E.C. Poggio, H.R. Quinn and S. Weinberg, Phys. Rev. D **13**, 1958 (1976).
- [19] M. Shifman, TPI-MINN-95/15-T (1995) [hep-ph/9505289], to appear in: Proc. Joint Meeting of the Int. Symp. on Particles, Strings and Cosmology & 19th Johns Hopkins Workshop on Current Problems in Particle Theory, Baltimore, Maryland, March 1995.